APPENDIX A-4: MEASUREMENT ACCURACY

Using Approximate Numbers To Express Measurement Accuracy

A typical measurement process will produce similar but not identical results for repeated measurements. Approximate numbers can be used to communicate this. For example, a set of 9 timing measurements might be:

7.2644 seconds
7.2592 seconds
7.2628 seconds
7.2646 seconds
7.2547 seconds
7.2672 seconds
7.2731 seconds
7.2639 seconds
7.2705 seconds

How should these measurements be summarized in a single reported value?

Significant-Digit Reporting:

Clearly the first two digits are dependable, or "significant" – they are the same in every case. Any report should obviously keep those values.

It is also clear that the last two digits, which vary almost randomly, contain no real information. These guesswork "insignificant" digits only confuse matters and should not be included in the reported value.

The middle digit is not random (it is always 5, 6, or 7), but is also not fully dependable. While it would be misleading to treat it just the same as the first two digits, something would be lost if nothing was said about this partially-significant digit. A common practice is to place any such transitional digit in parentheses, or to mark it in some other way.

Applying these significant-digit rules to the set of measurements listed above gives a summary value of:

7.2(6) seconds

Here is another example, using distances in meters:

126,402   126,328   126,475   126,510   126,306   126,392   126,422
126,287   126,573

The difficulty here is that the insignificant digits are to the left of the decimal point, so they cannot just be discarded. The solution is either to use different engineering units (kilometers instead of meters in this case) or to use scientific notation, which puts all digits except the leftmost one after the decimal point. For the measurements in this example, good summary statements would thus be either

126.(4) kilometers or

1.26(4) $\times$ **10**^{5} **meters**.

Averaging, then rounding, to determine the transitional digit:

Simply using counts of the digits in the transitional position will tend to slightly understate the value, since in about half of the cases the digit would be increased by one if the number were rounded to that place. Since the point of the transitional digit is to communicate the typical value, the best way to identify it is to either arrange the values in order and take the middle value (the kind of average called the *median*) or to divide the sum of all the values by the number of values (the kind of average called the *mean*). That average is then rounded to the precision of the transitional digit, which is thus (after the rounding) the final digit.

For example, consider this five-measurement set: *4.852 4.862 4.856 4.868 4.859* Even though the transitional third digit is 5 in a majority of cases, 6 is actually a better choice. This can be shown by either of these facts:

[i] The middle value when arranged by order of size is 4.859, which clearly rounds to 4.86

[ii] The mean value (the sum divided by 5) is 4.8592, which yields the same rounded result.

While use of an average is the best way to determine the transitional digit, other methods of summarizing measurements are usually employed when very careful computations are being made. Thus use of an informal estimate (which will seldom be off by more than one in the transitional place, which is not being seriously depended on anyway) is common.

Other Measurement-Summarizing Methods:

While significant-digit reporting is widely used and suffices for many purposes, people also use several other ways to communicate the accuracy of measurements. Generally, the method used is chosen to address the particular needs of the area. The most common alternative methods are:

[1] State the range in which all or some specific fraction of the measurements fall.

"*All measurements were between 7.2557 and 7.2731.*"

"*2/3 of the measurements fell between 127.2628 and 7.2705*"

"*The call length by which 95% of the service calls were completed was 7.2 minutes.*"

This method is appropriate when the fraction is predetermined (by a regulation on how to test, for example). It is common to use fractions such as 95% or 99% rather than the full range so that a few bad or non-typical values in a large set of measurements can be ignored.

[2] State the fraction of the measurements that fall within a specific range.

"*98.2% of the cylinders were within the tolerance range of 85.00 to 85.10 millimeters.*"

"*The fraction of service calls that met the company target of 3 minutes or less was 58%.*"

As the examples indicate, this method is used when the range is predetermined. It is particularly appropriate when, as in a typical manufacturing situation, all items outside the tolerance range are thrown away.

[3] State an average and a typical amount by which measurements deviate from that average.

"*7.2657 ± 0.0055 seconds*"

"*126,485 meters, with an average measurement error of ±108 meters*"

"*529.3 ± 12.4 (2σ)*"

The *average ± deviation* form should be used with care, since many measurements have greater deviations than the typical value, and several different ways of computing and expressing the deviation are used in different areas of work. While this approach is important in the theories of sampling and error propagation (where the form of typical deviation computed is a special "root mean square" average called the *standard deviation*, usually symbolized as the

Greek letter sigma, or σ), simpler methods of expressing accuracy will usually be more successful in areas where people have not been trained in these theories.

Problems:

[1] State an appropriate summarizing value for each of these sets of measurements, using significant digits:

[a]	*[b]*	*[c]*	*[d]*
92.154	88,329,364	23.97	0.00414
92.232	88,357,105	24.03	0.00449
92.188	88,334,286	23.95	0.00452
92.191	88,342,083	23.99	0.00473
92.157	88,339,271	24.01	0.00407
92.203	88,348,902	23.96	0.00426
92.151	88,351,283	23.97	0.00469
92.202	88,330,011	24.00	0.00480

[2] Find an example from a newspaper, magazine, or web site of a statement of measurement accuracy that:

[a] states the range over which a predetermined fraction of a measurement set falls.

[b] states the fraction of measurements that fall in a predetermined range.

Using Approximate Numbers To Express Measurement Accuracy – ANSWER KEY

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[a] The best summary value is 92.(2)

While it is tempting to use 1 as the transition digit because it occurs more frequently than 2 in that place, each of the listed values would round to 92.2, which is thus the best representative. However, use of a 1 would not be a significant misrepresentation, since the last digit is not being claimed as accurate.

[b] The best summary value is 8.83(4) $\times 10^7$

The average of these measurements is 88,341,538, which rounds to the value above. A listing in order of size of the values rounded to four digits (8833-8833-8833-8834-8834-8835-8835-8836) also indicates that the best transition digit is 4, since it occurs on both sides of the middle of the list.

[c] An appropriate summary value is 23.9(8), or else just 24.0

Even though the second digit changes, both it and the third digit are fully significant because the values vary by less than 0.1. It is just because the fractional part of the average value of 23.98 is so close to a power of ten (100 in this case) that the variation in the fourth digit causes the other digits to be changed by the effects of carrying. If the differences from 24.00 are listed (sorted by size they are –0.05, -0.04, -0.03, -0.03, -0.01, 0.00, +0.01, and +0.03), it is easier to see that the measurement varies by the amount that is characteristic of the last digit. An alternative in a case like this is to just use the transitional digit to compute a rounded value all of whose digits are significant.

[d] An appropriate summary values is 0.004(5)

The average of the values is 0.00446, so the values are centered on 0.0045, but there is so much variation that the fourth decimal place has little significance. However, even though using 0.004 without a transitional digit would not be wrong (since that rounded value promises only that the number is somewhere between 0.0035 and 0.0045), it does not convey the where the center of the measurement set was nearly as well as including the (5) does.

TEST – APPROXIMATE NUMBERS – *Answer Key*

Use nearest-neighbor rounding in all cases, including use of the even-number rule for breaking any exact ties.

[1-2] Round off each of the given numbers to three decimal places.

[1] 0.00027 **0.000**

[2] 0.0797 **0.080**

[3-4] Round off the given values as specified.

	Initial value	*Rounding Precision*	*Rounded Value*
[3]	5,742,597	50,000	**5,750,000**
[4]	0.082541	0.0001	**0.0825**

[5-6] Express the given values in engineering notation.

[5] 29,400,000 $\mathbf{2.94 \times 10^{7}}$

[6] 1.92×10^{19} $\mathbf{19.2 \times 10^{18}}$

[7-8] Express the given values in ordinary notation.

[7] 7.12 E 5 **712,000**

[8] 3.7×10^{-3} **0.0037**

[9-11] Convert the given values to the indicated engineering units.

[9] 152,900 watts = **152.9** kW

[10] 5×10^{8} bytes = **500** MB

[11] 0.00081 seconds = **810** μs

[12-13] Choose appropriate engineering units for the given values, and then express the values in those units.

[12] 12,600,000 watts = **12.6** **MW**

[13] 25×10^{-6} grams = **25** **μg**

[14] Round off the 1990 US census total family count of 64,517,947 to two digits and express it in scientific notation.

6.4×10^{7}

[15-17] State the smallest and largest actual values that would have been properly rounded to the given rounded values.

[15] 12.6 million **12.55 million** (smallest) **12.65 million** (largest)

[16] 4×10^{5} **3.5×10^{5}** (smallest) **4.5×10^{5}** (largest)

[17] 9.6 **9.55** (smallest) **9.65** (largest)

[18] Provide an appropriate summary value for this set of measurements.

9.6145 9.6392 9.6253 9.6321 9.6272 9.6407 9.6232 9.6380

9.6(3)

[19] If a summary measurement value is reported as 62.5(3), which of the following values would it be surprising to find in a small measurement set?

62.563 62.499 ***62.418*** 62.545 62.538

[20] In the sentence below, which numbers are approximate and which numbers are exact?

68% of the 84 million American males over the age of 15 have been married 1 time.

Approximate: *68%; 84 million* **Exact: *15; 1***

[21] A grain silo is being emptied by loading its contents into a truck. The truck has been rated by its manufacturer as having a capacity of 372 bushels, and the farmer is given that much credit for each truckload. If 29 trips are required to empty the silo, by how many bushels could the amount credited to the farmer be different from the correct amount without any misrepresentation in the manufacturer's rating?

The capacity was expressed to a precision of 1 bushel. This means that a rounding error of up to ½ bushel is still within the represented accuracy. It the actual truck capacity was 0.5 bushels different from the rated value, 29 trips would result in a total error of:

***29* × 0.5 = <u>14.5</u> bushels**

[22] The volume of a rectangular aquarium is the product of its width, its length, and its height.

[a] What is the range of legitimate values for the volume in cubic feet of a rectangular aquarium specified to be 0.5 feet wide, 1.2 feet long, and 1.0 feet high, with the understanding that these are all rounded numbers?

The minimum volume would result if each dimension were at the smaller end of the rounding range. In this case, that would be:

0.45 × 1.15 × 0.95 = 0.491625 ª <u>0.492</u> cubic feet

The maximum volume would result from the product of the corresponding numbers at the larger end of the rounding range, which would be:

0.55 × 1.25 × 1.05 = 0.721875 ª <u>0.722</u> cubic feet

[b] What percentages are the minimum and maximum values computed in [a] of the volume of an aquarium with exact 0.5-by-1.2-by-1.0 measurements?

The exact-case volume is $0.5 \times 1.2 \times 1.0 = 0.600$ *cubic feet, which means that the requested percentages are:*

Minimum: **0.492 / 0.600 = <u>82%</u>**

Maximum: **0.722 / 0.600 = <u>120%</u>**