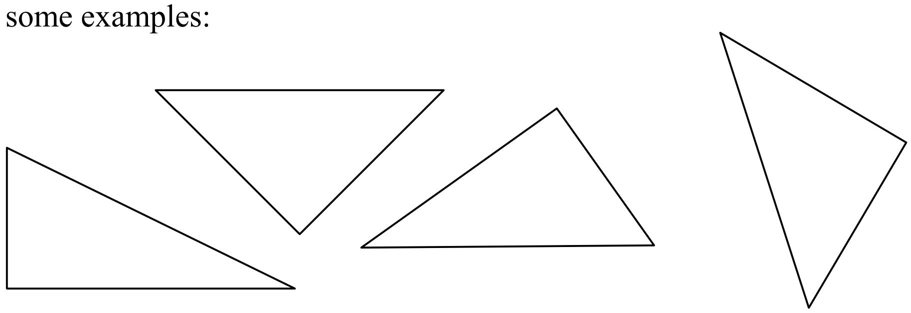
APPENDIX B-1: RIGHT TRIANGLES

RIGHT TRIANGLES

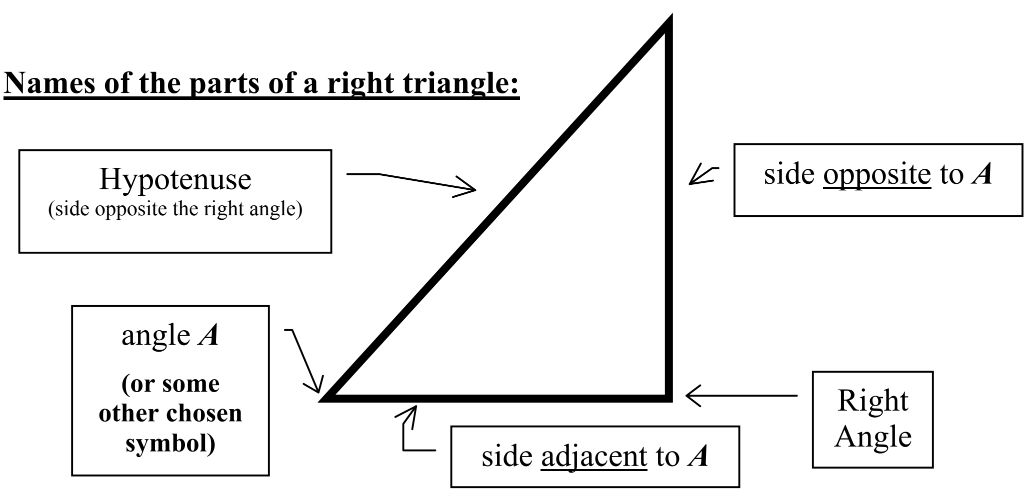
Many mathematical measurements of size, distance, and direction are based on ***triangles***, which are formed by the straight lines joining three points. Each of the three corners formed by the lines is called an ***angle*** of the triangle, and the lines between the corners are called ***sides*** of the triangle.

When two of the sides of a triangle are perpendicular to each other (such as when one is exactly vertical and the other is exactly horizontal, or when one meets the other like the sides of this sheet of paper), the triangle is called a ***right triangle***, and that angle is called a ***right angle***. Right triangles are especially important in both geometric theory and in practical measurement applications.

Because the right angle can be formed by sides that have any desired length, right triangles come in many shapes, sizes, and orientations. Here are some examples:

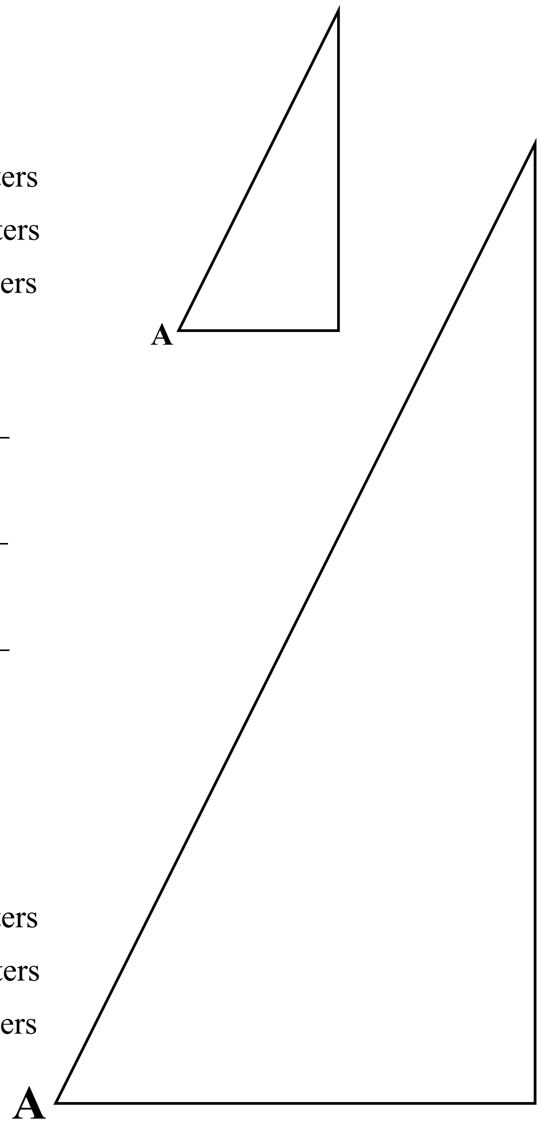


Names of the parts of a right triangle:



Exercise 1: For each of the examples shown above, choose one of the non-right angles to label as *A*, then mark the three sides of the triangle with *HYP*, *ADJ A*, and *OPP A* for the hypotenuse and the sides that are respectively adjacent to and opposite to your chosen A.

Exercise 2: **Measurements and ratios for two triangles that have the same shape:**



SMALL TRIANGLE

Sides of the small triangle:

opposite to A: _______ millimeters

adjacent to A: _______ millimeters

hypotenuse: _______ millimeters

ratios of small sides:

$\frac{\text{opposite to A}}{\text{hypotenuse}}$ = _________

$\frac{\text{adjacent to A}}{\text{hypotenuse}}$ = _________

$\frac{\text{opposite to A}}{\text{adjacent to A}}$ = _________

LARGE TRIANGLE

Sides of the large triangle:

opposite to A: _______ millimeters

adjacent to A: _______ millimeters

hypotenuse: _______ millimeters

ratios of large sides:

$\frac{\text{opposite to A}}{\text{hypotenuse}}$ = __________

$\frac{\text{adjacent to A}}{\text{hypotenuse}}$ = __________

$\frac{\text{opposite to A}}{\text{adjacent to A}}$ = __________

The most important *trigonometric functions* of an angle A, based on the ratios between the sides of a right triangle containing it, are:

sine of A (usually written as *sin A*) = $\frac{\text{side opposite to A}}{\text{hypotenuse}}$

cosine of A (usually written as *cos A*) = $\frac{\text{side adjacent to A}}{\text{hypotenuse}}$

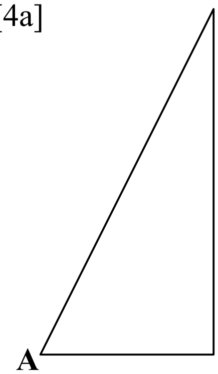
tangent of A (usually written as *tan A*) = $\frac{\text{side opposite to A}}{\text{side adjacent to A}}$

Exercise 3: Using your figures for the large triangle in Exercise 2, state the numerical values of the trigonometric ratios of the angle A in that case:

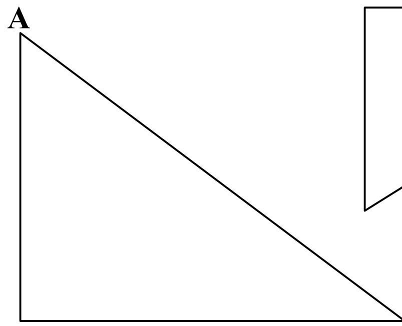
sin A = ____________ cos A = ____________ tan A = ____________

Exercise 4: For each right triangle below, find the specified trigonometric ratio (that is, the sine, cosine, or tangent of the angle labeled A for that triangle) by measuring the appropriate sides and then dividing according to the definitions above in order to compute the appropriate ratio.

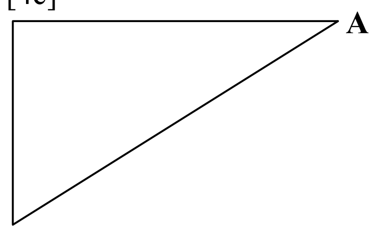
[4a]



[4b]



[4c]



sin A = _________ cos A = __________ tan A = __________

Exercise 5:

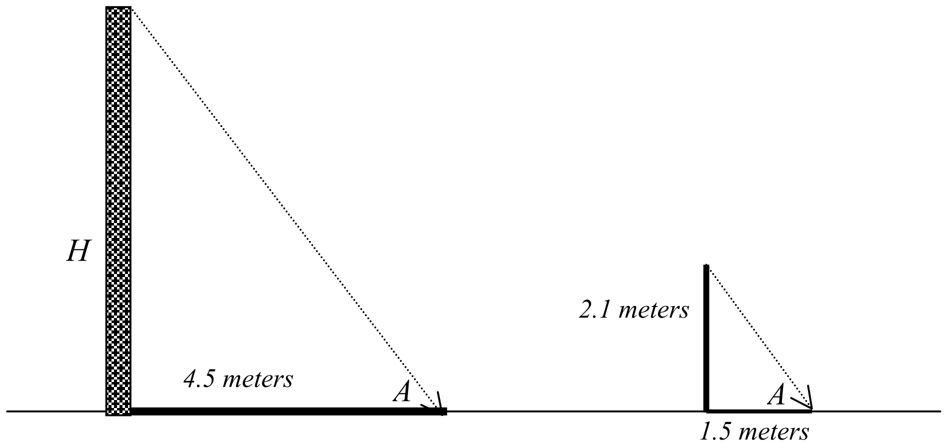
Measurements of parts of similar triangles can sometimes be used to determine the size of features that can not be measured directly. The diagram below illustrates one such situation. The high vertical wall shown on the left casts a shadow 4.5 meters long. At the same time the signpost on the right, which is 2.1 meters high, casts a shadow 1.5 meters long. The problem is to find the height of the wall from this information.

The sun is at the same angle *A* for both objects (since they are measured at the same time in nearby places), so that two similar right triangles are formed. This means that the ratios of corresponding sides are equal. Thus the height of the wall divided by the length of the wall's shadow will equal the height of the sign divided by the length of the sign's shadow. This is shown mathematically in the following equation, where *H* represents the height of the wall. This equation can be solved for *H* to compute the wall's height.

$$\frac{H}{4.5\ meters} = \frac{2.1\ meters}{1.5\ meters} = 1.4$$

$$\frac{H}{4.5\ meters} \cdot 4.5\ meters = 1.4 \cdot 4.5\ meters$$

$$H = 6.3\ meters$$



Note that the value 1.4 is the tangent of the angle A that the sun's rays make with level ground, since in both right triangles shown it is the ratio of the length of the side opposite A (the height) to the length of the side adjacent to A (the shadow length). While there is no need to use this fact in solving this particular problem, it could be used to derive a formula for this kind of problem, or to find the size of the angle A.

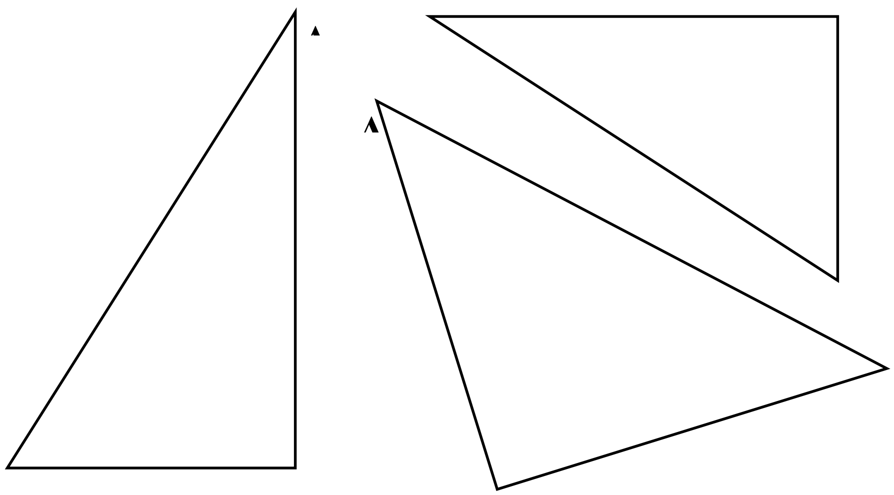
The formula would be: ***Height* = *shadowlength* â tan *A***

How to find the size of an angle from one of its trigonometric ratios will be explained in the next class period.

HOMEWORK

For all problems, show your work.

[1] For each of the three right triangles shown below, label these parts
- [a] the right angle
- [b] the hypotenuse
- [c] the side opposite to the angle labeled *A*
- [d] the side adjacent to the angle labeled *A*



[2] After making and recording any measurements needed in each case, compute
- [a] the tangent of angle A in Triangle 1
- [b] the cosine of angle A in Triangle 2
- [c] the sine of angle A in Triangle 3

Use a separate sheet of paper to record the needed measurements and show your work.

[3] Using one of the right triangles supplied by the teacher (or you can use one that you make yourself by cutting off a corner of a sheet of paper), determine and record these values:
- [a] the length of each side, to the nearest millimeter
- [b] the sine, cosine, and tangent of the smallest angle
- [c] the sine, cosine, and tangent of the other non-right angle

Use a separate sheet of paper to record the needed measurements and show your work.
Also bring the cut-out triangle to class – we will use it in discussing angle measurement.