APPENDIX C-1: MEASUREMENT CALIBRATION

CALIBRATION OF MEASUREMENT PROCESSES

The difference between any particular measurement and the true value for what is being measured is called the ***measurement error*** for that measurement. Thus if a measurement process yields a value of 78 millimeters for the length of a line that is actually 76.2 millimeters long, there is a measurement error of +1.8 millimeters. As explained below, the causes of measurement errors can be *systematic* or *random*.

When further computations are made based on measurements, any measurement errors will almost always cause some error in the final result of the computation. How sensitive a computation is to errors in the values used depends very much on the exact nature of the computation, and can (as we will see in a later lesson) often be predicted by examining a graph of what final results would be produced for different measurement values.

Systematic Errors

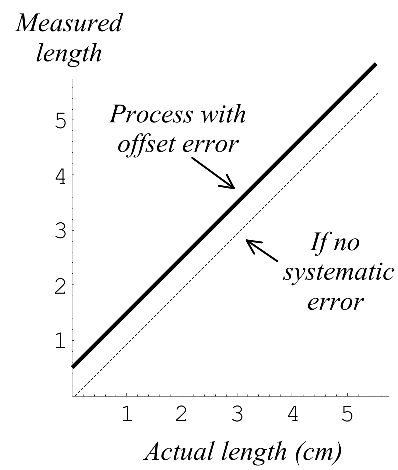
If the average value produced by repeating a measurement process is *consistently* larger or smaller than the true value for an object, this indicates that the process includes a ***systematic*** error. Such errors can often be corrected for by *calibrating* the process.

Examples of simple systematic errors. These three examples refer to the broken ruler shown below, in which the first half-centimeter has been cut off. (Assume that all measured objects are small enough that only a single ruler placement is used.)

Example A – **Offset:** In the first process, the broken ruler is used to measure length in centimeters. The measurements are made by lining up the low-number end of the ruler with one edge of the object, then directly reading off the ruler value at the object's other edge.

Consequences: *Because no allowance is being made for the fact that the ruler does not start at zero, all measurements made with this process will give values 0.5 cm longer than the true ones. An unchanging error of this kind (+0.5 centimeters in this case) is called an* ***offset error****.*

Correction procedure: **subtract the offset error from the uncorrected value.** Thus in this case 0.5 centimeters should be subtracted from each measurement.

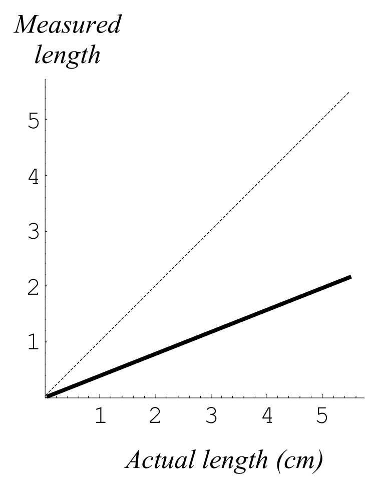


Process with offset error

Example B – **Scaling:** Someone intending to measure length in centimeters uses the wrong edge of the ruler, so that measurements reported as centimeters are actually made in inches. (This is perhaps too big an error to escape notice, but it makes a dramatic graph.)

Consequences: *Each reported length will be about 39% of the actual value (since $^{1}/_{2.54} \cong$ 0.39). Such a proportional error is called a* ***scaling error****.*

Correction procedure: **divide the uncorrected value by the *scale factor*** (that is, by the ratio of the measured values to the actual ones). In this case, each measurement should thus be divided by 0.3937 (or multiplied by 2.54) in order to give correct values in centimeters.

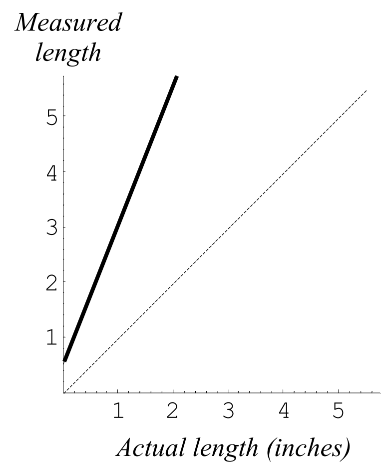


Process with scaling error

Example C – **Offset *and* scaling:** A person trying to measure lengths in inches uses the wrong edge of the broken ruler, and reports as inches measurements that are actually made with the ruler's centimeter scale (which also has the cut-off start point).

Consequences: *In this case, the number read off as a measurement will be 0.5 greater than would be correct for a centimeter measurement; but even a correct centimeter number is 2.54 times larger than the correct number for a measurement in inches.*

Correction procedure: **correct the value in separate offset and scale steps.** First subtract 0.5 to produce the correct length in centimeters (this is an offset correction). Then divide that result by 2.54 to get the correct length in inches (a scaling correction).



Process with offset *and* scaling errors

Note that in all the examples above, the graph that shows the relationship between the true value and the measured value is a *straight line*. The ***offset*** is shown on the graph by the *intercept* point of where the line crosses the vertical axis, and the ***scale factor*** is the *slope* of the line (the amount of vertical change for each unit of horizontal change). The equation of a line of this type is thus

$$\textbf{\textit{measurement}} = \textbf{\textit{actual value}} \times \textbf{\textit{scale factor}} + \textbf{\textit{offset}}$$

But we usually want to compute the actual values from measurements, not the other way around. We can do this by rearranging the above equation into the following **linear correction formula**:

$$\textbf{\textit{actual}} = \frac{\textbf{\textit{measurement}} - \textbf{\textit{offset}}}{\textbf{\textit{scale factor}}} = \frac{\textbf{\textit{measurement}} - \textbf{\textit{intercept}}}{\textbf{\textit{slope}}}$$

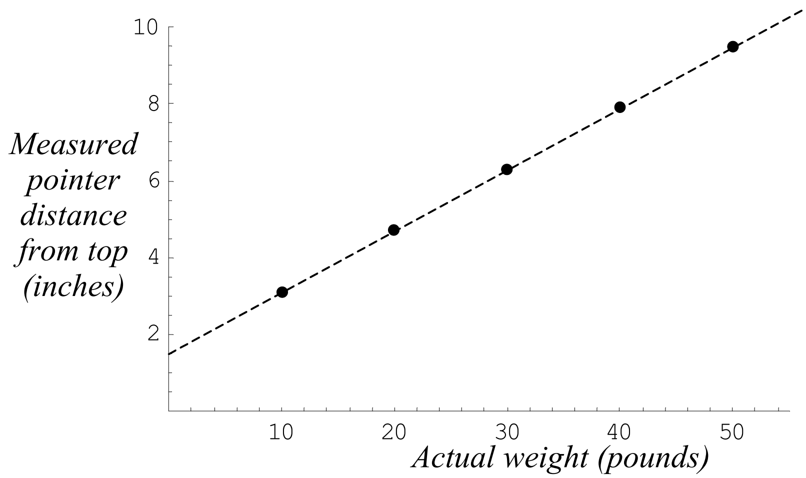
Measurement-Process Calibration

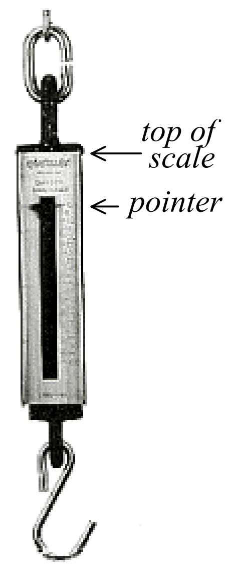
The standard way of investigating a measurement process is to use it to measure a set of differing objects whose true measurement values are already known. Typically the objects measured for calibration include ones that reflect the variety of sizes over which the measurement process will be used. These *reference measurements* can be used to form a graph of the type shown in the earlier examples.

Example D – ***Calibration***

In order to calibrate a spring scale of the kind shown, the distance of the pointer from the top of the scale is measured for a series of known weights. Here are the results, and the calibration graph:

Weight (in pounds):	10	20	30	40	50
Position (in inches):	3.1	4.7	6.3	7.9	9.5





The calibration points all fall on a line, so this measurement process is linear. The line's intercept is 1.5 inches (this is the *offset* of this process) and its slope (which is the *scale factor* of the process) is 0.16 inches per pound. Therefore the formula that provides a model for the measuring process is

distance from top $=$ ***actual weight*** $\times$ 0.16 inches/pound $+$ 1.5 inches

This modeling formula can be used to determine where the weight markings on the scale should be placed.

Even if the scale was not marked with weight values, knowing the offset and scale factor for the process would permit the construction of its linear correction formula (shown below), which could be used to calculate an object's weight from the pointer position that resulted when it was suspended from the scale.

$$\textit{\textbf{actual weight}} = \frac{\textit{\textbf{distance from top}} - 1.5 \text{ inches}}{0.16 \text{ inches/pound}}$$

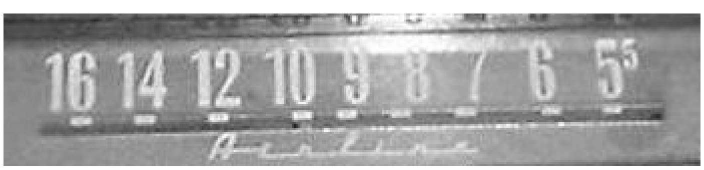
Exercise 1: What is the actual weight of an object which, when suspended from the scale calibrated above, moves the pointer to a position that is 4.3 inches from the top?

Using Calibration Measurements

Checking linearity: As shown in the example, the calibration graph can be used to test whether the measurement process is linear – are the points in a straight line? If they are, then the process is linear and the intercept and slope of the graphed line reveal, respectively, the offset and scale factor for this measurement process. (Note that at least three points are needed to test linearity.)

Linear calibration: The offset and scaling factor found for a linear process can in turn be used to make an appropriate linear correction formula that will convert any measurements made by this process into accurate values, removing the systematic error. (A process with no systematic error would have an offset of 0 and a scaling factor of 1, which means that its correction formula would leave the measurements unchanged.) Most measurement instruments have already had their settings adjusted in this way to remove systematic error, using procedures very similar to that used in the example.

Nonlinear calibration: Processes that are not linear can still be used to make accurate measurements. A common method is to use a calibration graph to convert the uncorrected measurements into accurate ones. Another method is to mark the indicator dial used to measure or control the process with calibrated reference values that match the nonlinearity of the process, permitting direct readout of accurate values.



Nonlinear indicator dial for selecting frequency in an old manually-tuned AM radio

Exercise 2: *Using the grid below (or graph paper of your own), determine by graphing whether the measurement process that produced the calibration data listed below can be well modeled as linear. If so, find approximate values for the offset and scale factor.*

Actual weight (pounds)	*Measured weight (pounds)*
10	*15*
30	*25*
60	*40*
100	*60*

<u>HOMEWORK –CALIBRATING MEASUREMENT PROCESSES</u>

Work all problems on a separate sheet of paper, showing your work in full.

[1] A person uses "heaping tablespoons" of ingredients in a situation where level tablespoons are called for.

[a] What kind of measurement error is being made: offset, scaling, or random?

[b] Based on your general knowledge, estimate the size of the error in this case.

[c] In a short statement, explain how you would calibrate this process.

[2] If a linear measurement process has an offset of 25.1 and a scale factor of 0.9, what measurement would be expected for an actual value of 45.7?

[3] If a linear measurement process has an offset of 72 and a scale factor of 1.5, what is the actual value corresponding to a measurement of 345?

[4] After a car has been driven a total of 5000 miles, its tires are changed from the factory-supplied ones that have 25-inch diameters to ones that have 23-inch diameters, with no adjustment of the odometer (the device on the dashboard that displays mileage based on the number of wheel revolutions). How can the actual mileage for this car now be computed from its odometer reading? *Hint: The smaller tires will revolve 25 times to travel the distance that the original tires covered in 23 revolutions (because tire circumference is proportional to diameter). Thus each new recorded mile will be due to only 0.92 miles of actual travel (since* $^{23}/_{25} = 0.92$*). Mileage readings on the odometer will thus show 8.7% more miles traveled after 5000 than is actually true (since* $^{25}/_{23} \simeq 1.087$*).*

[5] For each of the listed sets of calibration measurements, determine if the process that produced the data can be reasonably modeled as linear. Show your graphs.

Process A:

Actual value	10	20	30	40
Measured value	8	16	24	32

Process B:

Actual value	5	15	25	35
Measured value	16	28	40	52

Process C:

Actual value	15	30	45	60
Measured value	5	10	20	40

Process D:

Actual value	10	20	30	40
Measured value	100	80	60	40

Process E:

Actual value	25	50	100	200
Measured value	75	100	150	250

Process F:

Actual value	100	200	300	400
Measured value	75	225	300	325

Process G:

Actual value	25	50	75	100
Measured value	5	30	55	80

[6] For each of the linear processes in [5], estimate the offset and the scale factor from the graph.

[7] Given the calibration data shown for Process B, what ***measured*** value would be expected if the ***actual*** value is 30?

[8] Given the calibration data shown for Process D, what ***actual*** value would be implied if the ***measured*** value is 50?