APPENDIX C-2: RANDOM MEASUREMENT ERRORS

The Mathematical Effects of Random Measurement Errors

In addition to systematic errors (such as offset and scaling errors), which can be corrected for by calibration, all high-precision measurements include some unpredictable ***random*** errors that arise from the impossibility of exactly repeating a measurement process. The result is that any sequence of precise measurements, even of the same object, will *vary above and below the average value*.

Example of a series of measurement readings with random noise

Measurements:

13.6	13.4	13.3	13.2	13.5	13.4	13.3	13.5	13.1	13.4	13.8	13.3

Mean value:

13.4	*13.4*	*13.4*	*13.4*	*13.4*	*13.4*	*13.4*	*13.4*	*13.4*	*13.4*	*13.4*	*13.4*

Random Error:

+0.2	0.0	–0.1	–0.2	+0.1	0.0	–0.1	+0.1	–0.3	0.0	+0.4	–0.1

Such random errors cannot be corrected for by the calibration techniques that are used for systematic errors. On the other hand, in many situations the various random errors that are included in the measurements partially cancel each other out, unlike the consistent systematic errors.

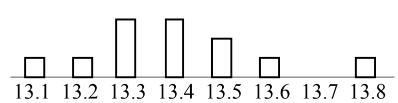
There are two important and useful mathematical methods for dealing with such random "noise" in measurements (in addition to the practical solution of using more stable measurement techniques or equipment, when that is feasible):

[1] **Determine how noisy a measurement process is**, so that users of its measurements can know the extent to which a measurement can be depended on. This is done by taking multiple measurements of the same thing and keeping track of how much the answers vary. The mathematical tools for noise measurement include "bell curve" graphs of how likely the various amounts of noise are and statistical definitions that can give numerical measures of the amount of noise in a measurement process.

"*All 12 measurements gave values between 13.1 and 13.8*"

"*2/3 of the 12 measurements fell between 13.3 and 13.5*"

"*The 12 measurements varied by up to 3% from their mean value*"



[2] **Combine repeated measurements** of the same thing by some form of *averaging* to produce values that have less noise than single measurements, and are thus more dependable. This is particularly useful during calibration processes or for other critical measurements, but can also be used whenever the increased stability is worth the cost of the extra measurements. There is a simple mathematical formula that predicts how many independent measurements must be averaged to produce a desired improvement in stability.

Types of averages

Mean:
The sum of the values divided by the number of values

Median:
Value in the middle if the values are sorted by size (if there are an even number of values, use the number halfway between the middle two)

Trimmed mean:
Discard some fraction of the values from the top and bottom of a sorted list, then compute the mean of the rest

Exercises on summarizing information in a measurement set

The voltage supplied by a battery was measured 12 times with a digital voltmeter, giving the following values (in volts):

1.536 1.583 1.551 1.526 1.565 1.544 1.559 1.530 1.548 1.557 1.577 1.564

[a] ***Tabulation***

Write each measurement in the appropriate subrange category, and count the number of entries in each category:

Measurement subrange	*Measurement instances*	*count*
1.520 ≤ measurement < 1.530		
1.530 ≤ measurement < 1.540		
1.540 ≤ measurement < 1.550		
1.550 ≤ measurement < 1.560		
1.560 ≤ measurement < 1.570		
1.570 ≤ measurement < 1.580		
1.580 ≤ measurement < 1.590		

[b] ***Averages***

The mean value of the measurements is __________ volts.

The median value of the measurements is ___________ volts.

The trimmed mean of the middle 8 measurements is ____________volts.

[c] ***Range of variation***

The lowest measurement number is _________. The highest is _________.

The *range* of the measurements (highest minus lowest) is _________ volts.

$^{2}/_{3}$ of values are between ___________ and ___________ volts.

The greatest absolute deviation from the mean value is __________ volts.

The greatest relative deviation from the mean value is ___________%.

HOMEWORK – Random Measurement Variation

Work all problems on a separate sheet of paper, showing your work in full.

[1] An object is weighed several times on an electronic scale, producing the following set of measurements. Describe the data set by answering the questions below.

Data values (in milligrams):								
5417	5407	5388	5370	5399	5384	5405	5366	5391

[a] Tabulate the values in appropriate measurement subranges that are each 10 milligrams wide.

[b] Compute the mean value of the measurements.

[c] Compute the median value of the measurements.

[d] Compute the trimmed mean of the middle 5 measurements.

[e] Compute the range of this set of measurements.

[f] State the relative greatest deviation from the mean as a percentage.

[2] Flip a coin 4 times and record your results. Do this 3 different times, noting the number of times it lands heads up for each set of 4 flips. The various values obtained by different people in the class get will be used in the next class.

[3] Produce a set of measurement data by picking two places at least a mile apart and measuring the distance with odometer readings from an automobile. Record the distance to a precision of 0.01 miles, making estimates as needed from the odometer (this will require that you estimate partial steps from the lowest digit. Repeat 4 times (two round trips), noting the variation in the results.